# $R^2$ curvature-squared corrections on drag force

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ABSTRACT: The effect of finite-coupling corrections to the drag force on a moving heavy quark in the Super Yang-Mills plasma is investigated. These corrections are related to curvature-squared corrections in the corresponding gravity dual. The results are compared with the dual gauge theory. It is shown that curvature-squared corrections affect the drag force. It is shown that corrections to the drag force depend on the velocity of the heavy quark. The diffusion coefficient of non-relativistic heavy quarks is calculated from the drag force. In addition, we also calculate the drag force on a moving heavy quark in the Gauss-Bonnet background.

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## 1. Introduction

The experiments of Relativistic Heavy Ion Collisions (RHIC), collisions of gold nuclei at 200 GeV per nucleon have produced a strongly-coupled quark-gluon plasma (QGP) [1]. The AdS/CFT correspondence [2 – 5] has yielded many important insights into the dynamics of strongly-coupled gauge theories. It has been used to investigate the hydrodynamical transport quantities in the various interesting strongly-coupled gauge theories where the perturbation theory is not applicable. Methods based on AdS/CFT relate the gravity in $AdS_5$ space to the conformal field theory on the 4-dimensional boundary. It has been shown that an AdS space with a black brane is dual to conformal field theory at finite temperature. Specially, dynamics of open strings on a $AdS_5$ black brane background is related to the quarks in the large N and large 't Hooft coupling limit of 4-dimensional the CFT dual theory at finite temperature. Then one can calculate energy loss of quarks to the surrounding strongly-coupled plasma.

One of the interesting properties of the strongly-coupled plasma at RHIC is jet quenching of partons produced with high transverse momentum. The jet quenching parameter controls the description of relativistic partons and it is possible to employ the gauge/gravity duality and determine this quantity at the finite temperature theories. There has been the AdS/CFT calculation of jet quenching parameter [10 – 19] and the drag coefficient which describes the energy loss for heavy quarks in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory [20 – 24].

The universality of the ratio of shear viscosity $\eta$ to entropy density $s$ [6 – 9] for all gauge theories with Einstein gravity dual raised the tantalizing prospect of a connection between string theory and RHIC. The results were obtained for a class of gauge theories whose holographic duals are dictated by classical Einstein gravity. But string theory contains higher derivative corrections from stringy or quantum effects, such corrections correspond

to $1/\lambda$ and $1/N$ corrections. In the case of $\mathcal{N} = 4$ super Yang-Mills theory, the gravity dual corresponds to type II$B$ string theory on $AdS_5 \times S^5$ background. The leading order corrections in $1/\lambda$ arises from the stringy corrections to the low energy effective action of type II$B$ supergravity, $\alpha'^3 R^4$. Such corrections to the ratio of shear viscosity $\eta$ to entropy density $s$ was calculated in [26, 27].

Recently, the calculation of $\frac{\eta}{s}$ for higher derivative gravity has been done by [28 – 30]. In [29], they compute the effect of general $R^2$ corrections to the gravitational action in AdS space and show that the conjecture low bound on the $\frac{\eta}{s}$ can be violated. The computations of this ratio in an effective five-dimensional setting have been discussed in [31]. Regarding [29] and motivated by the vastness of the string landscape [32], one can explore the modification of drag force on a moving heavy quark in the strongly-coupled plasma. We do not limit our study to specific known string theory corrections, and consider the generic higher derivative terms in the holographic gravity dual. In general, we do not know about forms of higher derivative corrections in string theory, but it has been known that by string landscape one expects that generic corrections can occur.

In this paper, we investigate finite-coupling corrections to the drag force on a moving heavy quark in the Super Yang-Mills plasma using AdS/CFT. These corrections are related to curvature-squared corrections in the corresponding gravity dual. It is shown that curvature-squared corrections affect the drag force. However, we do not predict effect of them on the CFT dual theory, because curvature-squared corrections are not the first higher derivative corrections in type II$B$ superstring theory. The corrections to the drag force depend on the velocity of heavy quark. It is shown in figure 1 that there is a critical velocity ($v_c$) such that for $v > v_c$ the corrections increase the drag force. This phenomena might be important because at $v_c$ curvature-squared corrections have the minimum effects to the drag force. The diffusion coefficient of non-relativistic heavy quark is calculated from the drag force. In addition, we also calculate the drag force on a moving heavy quark in the Gauss-Bonnet background. [41].

While this paper was in the final stages of preparation, it came to our attention that the drag force calculation for a Gauss-Bonnet brane has simultaneously been done in [41].

## 2. Set up

We study the gauge theory at finite temperature $T$ and assume the geometry has a black hole. In the gauge theory side, an external quark can be introduced by a string that has a single end point at the boundary and extends down to the horizon. One can consider the curvature-squared corrections on the AdS black brane solution of the following 5-dim action

$$S = \int d^5 x \sqrt{-g} \left[ \frac{R}{2\kappa} - \Lambda + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \right], \tag{2.1}$$

where $c_i$ are arbitrary small coefficients and the negative cosmological constant $\Lambda$ creates an AdS space with the radius

$$L^2 = \frac{-6}{\kappa\Lambda}. \tag{2.2}$$

For the well-known $\mathcal{N} = 4$ SU(N) SYM, $c_3$ term does not appear and one can use a field redefinition (as explained in [29]), but in general this does not need to be the case and one can find a theory with $c_3 > 0$ [36, 37]. It is important to point out that the first higher derivative correction in weakly curved type IIB backgrounds enters at order $R^4$, and not $R^2$, so we will not predict effect of curvature-squared corrections on the $\mathcal{N} = 4$ SU(N) SYM. The black brane solution of $AdS_5$ space with curvature-squared corrections is

$$ds^2 = -\left(\frac{r^2}{L^2}\right)f(r)dt^2 + \left(\frac{r^2}{L^2}\right)d\overrightarrow{x}^2 + \frac{1}{\left(\frac{r^2}{L^2}\right)f(r)}dr^2, \tag{2.3}$$

where

$$f(r) = 1 - \frac{r_0^4}{r^4} + \alpha + \gamma\frac{r_0^8}{r^8}. \tag{2.4}$$

and

$$\alpha = \frac{4\kappa}{3L^2}\left(2\left(5c_1 + c_2\right) + c_3\right), \quad \gamma = \frac{4\kappa}{L^2}c_3. \tag{2.5}$$

and $r$ denotes the radial coordinate of the black brane geometry and $t, \vec{x}$ label the directions along the boundary at the spatial infinity. In these coordinates the event horizon is located at $f(r_h) = 0$ where $r_h$ can be found by solving this equation. The boundary is located at infinity and the geometry will be as asymptotically AdS with the radius L. The temperature is given by

$$T = \frac{r_0}{\pi L^2}\left(1 + \frac{1}{4}\alpha - \frac{5}{4}\gamma\right). \tag{2.6}$$

The relevant string dynamics is captured by the Nambu-Goto action

$$S = -T_0 \int d\tau d\sigma \sqrt{-det g_{ab}}, \tag{2.7}$$

and $X^\mu(\sigma, \tau)$ is a map from the string world-sheet into the space-time. The coordinates $(\sigma, \tau)$ parameterize the induced metric $g_{ab}$ on the string world-sheet. Defining $\dot{X} = \partial_\tau X$, $X' = \partial_\sigma X$, and $V \cdot W = V^\mu W^\nu G_{\mu\nu}$ where $G_{\mu\nu}$ is the AdS black brane metric, we have

$$-g = -det g_{ab} = (\dot{X} \cdot X')^2 - (X')^2(\dot{X})^2. \tag{2.8}$$

One can make the static choice $\sigma = r, \tau = t$ and following [24, 33] focus on the dual configuration of the external quark moving in the $x^1$ direction on the plasma. The string in this case, trails behind its boundary endpoint as it moves at constant speed $v$ in the $x^1$ direction

$$x^1(r, t) = vt + \xi(r), \quad x^2 = 0, \quad x^3 = 0. \tag{2.9}$$

given this, one can find the lagrangian as the following

$$\mathcal{L} = \sqrt{-det g} = \sqrt{-G_{rr}G_{tt} - G_{rr}G_{xx}\,v^2 - G_{xx}G_{tt}\xi'^2}, \tag{2.10}$$

The equation of motion for $\xi$ implies that $\frac{\partial L}{\partial \xi'}$ is a constant. One can name this constant as $\Pi_\xi$ and it is found as follows

$$\Pi_\xi = \xi'\frac{G_{tt}G_{xx}}{\sqrt{-g}}, \tag{2.11}$$

The drag force that is experienced by the heavy quark is calculated by the current density for momentum along $x^1$ direction, $\pi_x^r$, where it is found as the following

$$\pi_x^r = -T_0\xi'\frac{G_{xx}G_{tt}}{(-det g_{ab})}, \tag{2.12}$$

and drag force is obtained

$$\frac{dp_1}{dt} = \sqrt{-det g_{ab}}\,\pi_x^r, \tag{2.13}$$

As a result, the drag force is easily simplified to

$$\frac{dp_1}{dt} = -T_0\Pi_\xi. \tag{2.14}$$

## 3. Drag force with the effect of curvature-squared corrections

One can replace the components of AdS black brane metric and derive the drag force. The lagrangian is calculated by (2.10) and one can find

$$\mathcal{L} = \sqrt{1 - \frac{v^2}{f(r)} + \frac{r^4}{L^4}f(r)\xi'^2}, \tag{3.1}$$

The constant of the motion is derived from equation of motion. $\xi'$ can also be found easily by deriving this constant. The result is as the following

$$\xi'^2 = \frac{\Pi_\xi^2\left(1 - \frac{v^2}{f(r)}\right)}{\frac{r^4}{L^4}f(r)\left(\frac{r^4}{L^4}f(r) - \Pi_\xi^2\right)}, \tag{3.2}$$

Near the horizon $r_h$, both numerator and denominator are positive for large $r$ and negative for small $r$. We are interested in a string that stretches from the boundary to the horizon. In such a string, $\xi'^2$ remains positive everywhere on the string. As a result both numerator and denominator change sign at the same point and with this condition, one can find the constant of the motion. The numerator changes sign at the following point

$$r_{critical}^4 = \frac{r_0^4}{2(1 + \alpha - v^2)} + \frac{r_0^4\sqrt{1 - 4\gamma(1 + \alpha - v^2)}}{2(1 + \alpha - v^2)}, \tag{3.3}$$

and it is clear that in the case of $\alpha = 0 = \gamma$, we will notice the result of AdS black brane in [24, 33]. The denominator changes sign at the same point. As a result, the constant of the motion is derived as follows

$$\Pi_\xi^2 = \frac{v^2 r_0^4}{2L^4(1 + \alpha - v^2)}\left(1 + \sqrt{1 - 4\gamma(1 + \alpha - v^2)}\right). \tag{3.4}$$

It is straightforward to find $\xi'$. The result is a lengthy equation. The coefficients of $\alpha$ and $\gamma$ are small and one can expand $\xi'$ to obtain the leading order terms

$$\xi' = \frac{L^2 r_0^2 v}{r_0^4 - r^4} + \alpha\left(\frac{L^2 r_0^2 v}{r_0^4 - r^4}\right)\left(\frac{3r^4 - r_0^4}{2(r_0^4 - r^4)}\right)$$
$$+ \gamma\left(\frac{L^2 r_0^2 v}{r_0^4 - r^4}\right)\left(\frac{r^4 r_0^4(1 - 2v^2) + r_0^8(v^2 - 2) + r^8(v^2 - 1)}{2r^8(r^4 - r_0^4)}\right) + \dots. \tag{3.5}$$
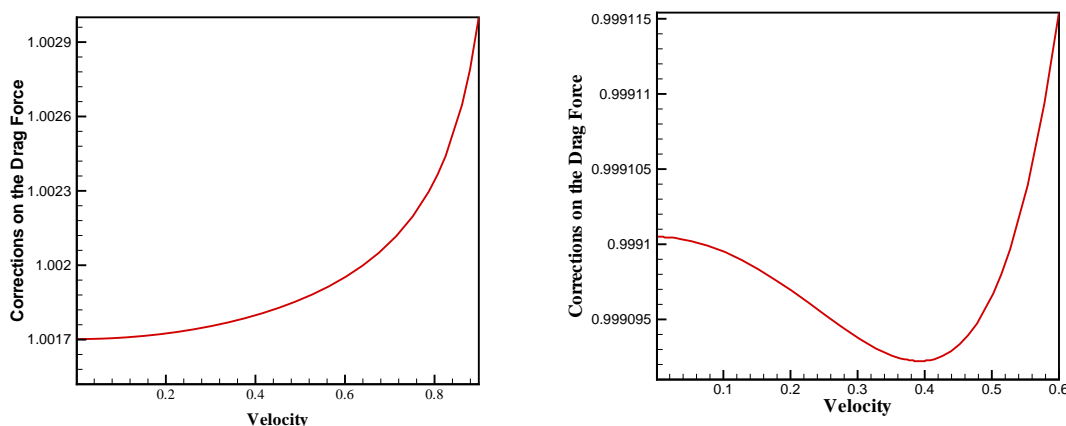
**Figure 1:** The corrections to the drag force versus the velocity of the heavy quark with fixed small values of $\alpha$ and $\gamma$. Left: $\alpha = -0.0005$ and $\gamma = +0.0006$ so $T_{R^2} < T_{\mathcal{N}=4}$. Right: $\alpha = -0.0005$ and $\gamma = -0.0007$ so $T_{R^2} > T_{\mathcal{N}=4}$.

The first term is the exact result in [24, 33]. By integrating the above terms, one can calculate $\xi$, too.

Finally the drag force on a moving heavy quark through the strongly-coupled with the effect of curvature-squared corrections is

$$\frac{dp_1}{dt} = -T_0 \left( \frac{v r_0^2}{\sqrt{2} L^2} \right) \left( \frac{1 + \sqrt{1 - 4\gamma(1 + \alpha - v^2)}}{(1 + \alpha - v^2)} \right)^{1/2}, \quad (3.6)$$

We rewrite the drag force in terms of parameters of the strongly-coupled plasma. The tension of the string is $T_0 = -\frac{1}{2\pi\alpha'}$ and temperature of the plasma is given by (2.6). It is easy to find the parameters of drag force in terms of temperature and coupling of the plasma. Using the above result, the drag force in terms of the temperature and coupling of the plasma is given by

$$(F_{\text{drag}})_{R^2} = -\frac{\pi T^2 \sqrt{g_{\text{YM}}^2 N}}{2} \left( \frac{v}{\sqrt{1-v^2}} \right) \sqrt{\left( \frac{1 + \sqrt{1 - 4\gamma(1 + \alpha - v^2)}}{2 \left(1 + \frac{\alpha}{4} - \frac{5\gamma}{4}\right)^4 \left(1 + \frac{\alpha}{1-v^2}\right)} \right)}. \quad (3.7)$$

We have considered the effect of curvature-squared corrections on the AdS black brane metric in (3.7). This equation describes the drag force on a moving heavy quark in the strongly-coupled supersymmetric Yang-Mills plasma. The drag force on a moving heavy quark in the dual gauge theory is derived from (3.7) if we neglect the effect of curvature-squared corrections by plugging $\alpha = \gamma = 0$ in (3.7).

The effect of curvature-squared corrections in (3.7) appears in the square-root.

We consider the square-root in (3.7) as the corrections to the drag force. As we mentioned in (2.5), parameters $\alpha$ and $\gamma$ are small constants. One can discuss the corrections to the drag force for different values of $\alpha$ and $\gamma$. As an example, we have plotted in figure

1 the corrections to the drag force versus the velocity of the heavy quark with fixed small values of $\alpha$ and $\gamma$. In the left plot of figure 1, $\alpha = -0.0005$ and $\gamma = +0.0006$ so $T_{R^2} < T_{\mathcal{N}=4}$ where $T_{\mathcal{N}=4}$ is the temperature of $\mathcal{N} = 4$, SYM plasma and $T_{R^2}$ is the temperature of SYM plasma with the effect of curvature-squared corrections, respectively. As it is clear from the left plot, the corrections to the drag force are increased monotonically with increasing the velocity of the heavy quark and the corrections to the drag force are larger than $\mathcal{N} = 4$ case. By studying the derivative of square-root in (3.7), one can see that it can vanish. This can be observed in the right plot of figure 1. In this plot, $\alpha = -0.0005$ and $\gamma = -0.0007$ so $T_{R^2} > T_{\mathcal{N}=4}$. In this case, the corrections to the drag force are smaller than $\mathcal{N} = 4$ case. As it is obviously seen from this plot, there is a critical velocity ($v_c$) such that for $v > v_c$ the corrections increase the drag force. The critical velocity and the condition on the parameters of $\alpha$ and $\gamma$ can be found by studying the derivative of square-root in (3.7). This phenomena might be important because at $v_c$ the curvature-squared corrections have the minimum effects on the drag force.

As a result, finite-coupling corrections affect the drag force on a moving quark in the strongly-coupled plasma and depend on the details of curvature-squared corrections in the corresponding gravity dual. The drag force can be larger than or smaller than that in the infinite-coupling case. We should emphasize that from our results one cannot predict a result for $\mathcal{N} = 4$ SYM because the first higher derivative correction in weakly curved type IIB backgrounds enters at order $R^4$, and not $R^2$.

## 4. $R^2$ corrections on the diffusion coefficient of heavy quark in the SYM plasma

The diffusion coefficient is a fundamental parameter of plasma at RHIC for heavy quarks. It is known that the knowledge of the viscous drag is equivalent to the knowledge of diffusion coefficient of quark. The diffusion coefficient of non-relativistic heavy quarks could be found from the drag force. We follow this approach in the case of $\mathcal{N} = 4$ Super Yang-Mills theory.

The drag force on a heavy quark moving through the dual gauge theory was calculated in [24, 33] as follows

$$(F_{\mathrm{drag}})_{\mathcal{N}=4} = \frac{\pi \sqrt{g_{\mathrm{YM}}^2 N}}{2} T^2 \frac{v}{\sqrt{1-v^2}} \, . \tag{4.1}$$

The relaxation time can be derived from the above equation as the following

$$t_{\mathcal{N}=4} = \frac{2 \, m}{\pi \, T^2 \, \sqrt{g_{\mathrm{YM}}^2 N}} . \tag{4.2}$$

The diffusion coefficient is related to the temperature of the plasma $T$, the heavy quark mass $m$ and the relaxation time $t_D$ as $D = \frac{T}{m} t_D$. It is straightforward to obtain the diffusion coefficient in the case of $\mathcal{N} = 4$ SYM plasma

$$D_{\mathcal{N}=4} = \frac{2}{\pi \, T \, \sqrt{g_{\mathrm{YM}}^2 N}} \, . \tag{4.3}$$

This result has been achieved with a different approach in [25], for non-relativistic heavy quarks. Using the above approach, we can obtain the diffusion coefficient with the effect of finite-coupling corrections. If we neglect the squared-velocity terms of (3.7), the drag force for non-relativistic heavy quarks is given by

$$(F_{\text{drag}})_{R^2} = -\frac{\pi\, p\, T^2\, \sqrt{g_{\text{YM}}^2 N}}{2\, m\sqrt{2}\, \left(1 + \frac{\alpha}{4} - \frac{5\gamma}{4}\right)^2} \left(\frac{1 + \sqrt{1 - 4\gamma(1+\alpha)}}{(1+\alpha)}\right)^{1/2}. \tag{4.4}$$

The relaxation time of moving heavy quark can be calculated from the drag force. According to the above discussion, one can obtain the diffusion coefficient with the effect of finite-coupling corrections as follows

$$D = \frac{2\sqrt{2}\, \left(1 + \frac{\alpha}{4} - \frac{5\gamma}{4}\right)^2}{\pi\, T\, \sqrt{g_{\text{YM}}^2 N}} \left(\frac{(1+\alpha)}{1 + \sqrt{1 - 4\gamma(1+\alpha)}}\right)^{1/2}. \tag{4.5}$$

where $\alpha$ and $\gamma$ are small constants. We can expand the above equation and keep the leading order terms, the result would be

$$D = \frac{2}{\pi\, T\, \sqrt{g_{\text{YM}}^2 N}} \{1 + \alpha - 2\gamma\}. \tag{4.6}$$

We find that finite-coupling corrections affect the result of the dual gauge theory and one can argue about different signs of $\alpha$ and $\gamma$. Also the rate of change of the mean square transverse momentum of a non-relativistic heavy quark will be changed to

$$\frac{d}{dt}\langle(\vec{p}_\perp)^2\rangle = \frac{4\,T^2}{D}. \tag{4.7}$$

Jet quenching parameter, $\hat{q}$, as defined in [35], can be found by dividing (4.7) to the velocity of the quark. Regarding the discussion of [25] about the mass of heavy quark, the relaxation time $t_D = \frac{m}{T}D$ must be larger than the inverse temperature

$$t_D \gg \frac{1}{T}, \tag{4.8}$$

which leads to

$$m \gg \frac{\pi\, T\, \sqrt{g_{\text{YM}}^2 N}}{2\,(1 + \alpha - 2\,\gamma)}. \tag{4.9}$$

One can tune mass by changing $\alpha$ and $\gamma$.

## 5. Gauss-Bonnet gravity background

We study the black holes with higher derivative curvature in the AdS space. In five dimensions, the most general theory of gravity with quadratic powers of curvature is Einstein-Gauss-Bonnet (EGB) theory. The exact solutions and thermodynamic properties of the

black brane in Gauss-Bonnet gravity were discussed in [38–40]. Authors in [28, 30] showed that for a class of CFTs with Gauss-Bonnet gravity dual, the ratio of shear viscosity to entropy density could violate the conjectured viscosity bound. The computations of this ratio in an effective five-dimensional setting have been discussed in [31]. We try to understand more about the drag force on a moving heavy quark in the boundary gauge theory by string trailing in the Gauss-Bonnet gravity.

The black brane solution in this geometry is given by

$$ds^2 = -a\,\frac{r^2}{L^2}\,h(r)\,dt^2 + \frac{dr^2}{\frac{r^2}{L^2}h(r)} + \frac{r^2}{L^2}\,d\vec{x}^2, \tag{5.1}$$

where

$$h(r) = \frac{1}{2\lambda_{\text{GB}}}\left[1 - \sqrt{1 - 4\lambda_{\text{GB}}\left(1 - \frac{r_+^4}{r^4}\right)}\right]. \tag{5.2}$$

In (5.1), $a$ is an arbitrary constant which specifies the speed of light of the boundary gauge theory and we choose it to be unity. As a result at the boundary, where $r \to \infty$,

$$h(r) \to \frac{1}{a}, \quad a = \frac{1}{2}\left(1 + \sqrt{1 - 4\lambda_{\text{GB}}}\right). \tag{5.3}$$

We assume $\lambda_{\text{GB}} \leq \frac{1}{4}$, the reason is that beyond this point there is no vacuum AdS solution and one cannot have a conformal field theory at the boundary. The curvature singularity for $\lambda \geq 0$ occurs at $r = 0$ and for $\lambda \leq 0$ the curvature singularity is located at $r = r_+\left(1 - \frac{1}{4\lambda_{\text{GB}}}\right)^{-1/2}$. The temperature is given by

$$T = \frac{\sqrt{a}\,r_+}{\pi\,L^2}. \tag{5.4}$$

Using (5.1) the lagrangian would become

$$\mathcal{L} = \sqrt{a - \frac{v^2}{h(r)} + a\frac{r^4}{L^4}h(r)\xi'^2}, \tag{5.5}$$

and the constant of the motion is derived from equation of motion. One may solve the relation for $\xi'$, the result is as the following

$$\xi'^2 = \frac{\Pi_\xi^2\left(a - \frac{v^2}{h(r)}\right)}{a\frac{r^4}{L^4}h(r)\left(a\frac{r^4}{L^4}h(r) - \Pi_\xi^2\right)}. \tag{5.6}$$

As before, we look for the string that stretches from boundary to horizon. Then numerator and denominator change sign at the same value

$$r_{\text{critical}} = \frac{\sqrt{a}r_+}{(a(a - v^2) + \lambda_{\text{GB}}v^4)^{\frac{1}{4}}}, \tag{5.7}$$

with this condition, the constant of the motion is derived from denominator

$$\Pi_\xi = \left(\frac{r_+^2 a}{L^2}\right)\frac{v}{\sqrt{a(a - v^2) + \lambda_{\text{GB}}v^4}}, \tag{5.8}$$

with this result, one can find the drag force from the above equations. Following (2.14), the drag force is derived from (5.8). The result is

$$\frac{dp_1}{dt} = -T_0 \left( \frac{r_+^2 a}{L^2} \right) \frac{v}{\sqrt{a(a - v^2) + \lambda_{\text{GB}} v^4}}.$$ (5.9)

We recall two useful formulas

$$L^4 = g_{\text{YM}}^2 N \alpha'^2, \quad T = \frac{r_+ \sqrt{a}}{\pi L^2},$$ (5.10)

where T is Hawking temperature or temperature of the plasma. Plugging these relations into the drag force leads to the final result for the drag force in the Gauss-Bonnet background

$$(F_{\text{drag}})_{\text{GB}} = -\frac{\pi \sqrt{g_{\text{YM}}^2 N}}{2} T^2 \frac{v}{\sqrt{a(a - v^2) + \lambda_{\text{GB}} v^4}}.$$ (5.11)

In this section, we considered the curvature-squared corrections and found the drag force in the Gauss-Bonnet gravity. Now, we compare this result with the drag force in AdS gravity where we know the dual gauge theory exactly. The drag force on a moving heavy quark through a thermal state of SU(N) of $\mathcal{N} = 4$ SYM theory is given by (4.1). One can compare the drag force in the Gauss-Bonnet background, $(F_{\text{drag}})_{\text{GB}}$ with the drag force in the case of $\mathcal{N} = 4$ SYM theory ,$(F_{\text{drag}})_{\mathcal{N}=4}$ as the following

$$\begin{aligned}
\frac{(F_{\text{drag}})_{\text{GB}}}{(F_{\text{drag}})_{\mathcal{N}=4}} &= \frac{\sqrt{1 - v^2}}{\sqrt{a(a - v^2) + \lambda_{\text{GB}} v^4}} \\
&= \frac{\sqrt{2}}{\sqrt{1 - 2\lambda_{\text{GB}}(1 + v^2) + \sqrt{1 - 4\lambda_{\text{GB}}}}}.
\end{aligned}$$ (5.12)

where we have used the definition of $a$ in the equation (5.3).

It would be interesting to find values of $\lambda_{\text{GB}}$ where the drag forces in two theories have the same value. It is clear from the denominator of (5.12) that at $\lambda_{\text{GB}} = 0$, the denominator is $\sqrt{2}$ and as a result the fraction of (5.12) will be unity! It is acceptable, because at this value all curvature-squared terms will be zero. An important point is that this value of $\lambda_{\text{GB}}$ is independent of velocity of heavy quark and we will not expect a critical velocity. One can obtain from the denominator of (5.12) that if $\lambda_{\text{GB}} > 0$ the drag force on a moving heavy quark in the Gauss-Bonnet gravity will be larger than $\mathcal{N} = 4$ case. Also for $\lambda_{\text{GB}} < 0$, the drag force is smaller than $\mathcal{N} = 4$ case.

The above discussion for $\lambda_{\text{GB}}$ is valid for the non-relativistic heavy quarks, where we neglect the squared-velocity term in the (5.12). The relaxation time and diffusion coefficient of the non-relativistic heavy quark moving through the strongly-coupled plasma in the Gauss-Bonnet gravity can be obtained by following the calculations of previous section.

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